Free-Riding on Enforcement in the WTO

Online Appendix – Theory

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A.1 Proofs for Results in the Paper

We assume that each player i's type in period t, α_{it}, is independently and identically distributed across players and time. Additionally, we assume that the common distribution function F has full support on interval $[\alpha_L, \alpha_H]$, where $0 \leq \alpha_L < \frac{c}{\tau_i} < \alpha_H$ for every i.

Let ρ_{-i} denote player i's belief about the probability that no other player will submit the dispute to the WTO, and hence that the game will proceed to period $t+1$. Let V_i denote player i's continuation value—her *ex ante* expected utility from playing the game. Because type α_{it}, is independently and identically distributed across time, neither ρ_{-i} nor V_i are a function of time.

Given the per period payoffs defined in the paper and conditional on reaching period t, player i's expected utility functions in period t are:

$$\begin{aligned} EU_{it}\left(\text{file}|\alpha_{it},\tau_i\right) &= \frac{\delta}{1-\delta}\left(r+b\right)\tau_i - c \\ EU_{it}\left(\text{don't file}|\alpha_{it},\tau_i\right) &= -\alpha_{it}\tau_i + \left(1-\rho_{-i}\right)\frac{\delta}{1-\delta}r\tau_i + \rho_{-i}\delta V_i \end{aligned}$$

Equilibrium Behavior

Proof of Proposition 1. Player i has incentive to file iff:

$$\begin{aligned} & \frac{\delta}{1-\delta}\left(r+b\right)\tau_i - c \geq -\alpha_{it}\tau_i + \left(1-\rho_{-i}\right)\frac{\delta}{1-\delta}r\tau_i + \rho_{-i}\delta V_i \\ \Leftrightarrow \quad & \alpha_{it} \geq \frac{c}{\tau_i} - \frac{\delta}{1-\delta}b - \rho_{-i}\frac{\delta}{1-\delta}r + \frac{\delta\rho_{-i}}{\tau_i}V_i \equiv \overline{\alpha}_i \end{aligned} \tag{1}$$

Equilibrium behavior is therefore monotonic: high types ($\alpha_{it} > \overline{\alpha}_i$) will file, and low types ($\alpha_{it} < \overline{\alpha}_i$) will not file. So player i's best response function can be characterized by the value of $\overline{\alpha}_i$ defined in equation (1).

Let ρ_i denote the *ex ante* probability that player i does not file in a given time period. Then:

$$\rho_i = \Pr\left(\alpha_{it} < \overline{\alpha}_i\right) = F\left(\overline{\alpha}_i\right)$$

Let ρ denote the *ex ante* probability that no player files in a given time period, and ρ_{-i} denote the *ex ante* probability that no player besides i files in a given time period (per the description above). Then:

$$\rho = \prod_{k=1}^{n}\rho_k = \prod_{k=1}^{n}F\left(\overline{\alpha}_k\right) \quad \text{and} \quad \rho_{-i} = \prod_{j\neq i}\rho_j = \frac{\prod_{k=1}^{n}F\left(\overline{\alpha}_k\right)}{F\left(\overline{\alpha}_i\right)} = \frac{\rho}{\rho_i}$$

Also note that $\rho = \rho_i \rho_{-i}$.

In an interior equilibrium—an equilibrium in which $\overline{\alpha}_i \in [\alpha_L, \alpha_H]$ for all i—player i's continuation value is therefore:

$$\begin{aligned} V_i &= \int_{\alpha_L}^{\overline{\alpha}_i} \left[-\alpha\tau_i + (1-\rho_{-i})\frac{\delta}{1-\delta} r\tau_i + \rho_{-i}\delta V_i\right] f(\alpha)\, d\alpha + \int_{\overline{\alpha}_i}^{\alpha_H} \left[\frac{\delta}{1-\delta}(r+b)\tau_i - c\right] f(\alpha)\, d\alpha \\ &= \rho_i \left[(1-\rho_{-i})\frac{\delta}{1-\delta} r\tau_i + \rho_{-i}\delta V_i\right] + (1-\rho_i)\left[\frac{\delta}{1-\delta}(r+b)\tau_i - c\right] - \tau_i \int_{\alpha_L}^{\overline{\alpha}_i} \alpha f(\alpha)\, d\alpha \qquad (2) \end{aligned}$$

Manipulating equation (2) to isolate V_i yields:

$$V_i = \frac{1}{1-\delta\rho}\left[(1-\rho)\frac{\delta}{1-\delta} r\tau_i - (1-\rho_i)\left(c - \frac{\delta}{1-\delta} b\tau_i\right) - \tau_i \int_{\alpha_L}^{\overline{\alpha}_i} \alpha f(\alpha)\, d\alpha\right] \qquad (3)$$

Substituting equation (3) into equation (1) yields:

$$\begin{aligned} \overline{\alpha}_i &= \frac{c}{\tau_i} - \frac{\delta}{1-\delta} b - \rho_{-i}\frac{\delta}{1-\delta} r \\ &\quad + \frac{\delta\rho_{-i}}{\tau_i(1-\delta\rho)}\left[(1-\rho)\frac{\delta}{1-\delta} r\tau_i - (1-\rho_i)\left(c - \frac{\delta}{1-\delta} b\tau_i\right) - \tau_i \int_{\alpha_L}^{\overline{\alpha}_i} \alpha f(\alpha)\, d\alpha\right] \qquad (4) \end{aligned}$$

If we manipulate equation (4), we can see that cutpoint $\overline{\alpha}_i$ is implicitly defined by the following function:

$$\Psi^i \equiv \overline{\alpha}_i (1-\delta\rho) - (1-\delta\rho_{-i})\left(\frac{c}{\tau_i} - \frac{\delta}{1-\delta} b\right) + \delta\rho_{-i}\left(r + \int_{\alpha_L}^{\overline{\alpha}_i} \alpha f(\alpha)\, d\alpha\right) = 0 \qquad (5)$$

To see that this function can generate an interior equilibrium, note that:

$$\Psi^i_{\overline{\alpha}_i} = \overline{\alpha}_i \left[-\delta\rho_{-i} f(\overline{\alpha}_i)\right] + (1-\delta\rho) + \delta\rho_{-i}\overline{\alpha}_i f(\overline{\alpha}_i) = 1 - \delta\rho > 0$$

Because Ψ^i is strictly increasing in $\overline{\alpha}_i$, if there exists a value $\overline{\alpha}_i$ that satisfies $\Psi^i(\overline{\alpha}_i) = 0$, this value is unique. Consider the value of function Ψ^i in the limit as δ becomes arbitrarily small:

$$\lim_{\delta \to 0} \Psi^i = \overline{\alpha}_i - \frac{c}{\tau_i}$$

We can therefore identify the equilibrium cutpoint as δ becomes arbitrarily small:

$$\lim_{\delta \to 0} \Psi^i = 0 \quad \Leftrightarrow \quad \lim_{\delta \to 0} \overline{\alpha}_i = \frac{c}{\tau_i}$$

Recall that by assumption, $\frac{c}{\tau_i} \in (\alpha_L, \alpha_H)$ for every i and α has full support over $[\alpha_L, \alpha_H]$. So player i has a unique interior cutpoint, $\overline{\alpha}_i \in (\alpha_L, \alpha_H)$, for small $\delta > 0$.

Since this argument holds for an arbitrary player i, there exists a Bayesian Nash equilibrium in which equilibrium strategies are implicitly defined by the system of n equations with n endogenous variables $\overline{\alpha} = (\overline{\alpha}_1, \overline{\alpha}_2, \ldots \overline{\alpha}_n)$:

$$\begin{aligned} \Psi^1(\overline{\alpha}) &= 0 \\ \Psi^2(\overline{\alpha}) &= 0 \\ \ldots & \ldots \ldots \\ \Psi^n(\overline{\alpha}) &= 0 \end{aligned}$$

□

Intermediate Results

Note that Ψ^i is continuously differentiable in all of its arguments. In particular,

$$\Psi^i_{\overline{\alpha}_i} = \overline{\alpha}_i \left[-\delta\rho_{-i} f(\overline{\alpha}_i)\right] + (1-\delta\rho) + \delta\rho_{-i}\overline{\alpha}_i f(\overline{\alpha}_i) = 1 - \delta\rho > 0$$

Note that: $\lim_{\delta\to 0} \Psi^i_{\overline{\alpha}_i} = 1$.

By manipulating equation (5) and using substitutions for the ρ-values, we can show that:

$$\Psi^i = \overline{\alpha}_i (1 - \delta\rho_j\rho_{-j}) - \left(\frac{c}{\tau_i} - \frac{\delta}{1-\delta} b\right) + \delta\left(\frac{\rho_j\rho_{-j}}{\rho_i}\right)\left(\frac{c}{\tau_i} - \frac{\delta}{1-\delta} b + r + \int_{\alpha_L}^{\overline{\alpha}_i} \alpha f(\alpha)\, d\alpha\right)$$

This allows us to see that the derivative of Ψ^i with respect to $\overline{\alpha}_j$ (for $j \neq i$) is:

$$\begin{aligned} \Psi^i_{\overline{\alpha}_j} &= \overline{\alpha}_i \left[-\delta f(\overline{\alpha}_j)\rho_{-j}\right] + \delta f(\overline{\alpha}_j)\left(\frac{\rho_{-j}}{\rho_i}\right)\left(\frac{c}{\tau_i} - \frac{\delta}{1-\delta} b + r + \int_{\alpha_L}^{\overline{\alpha}_i} \alpha f(\alpha)\, d\alpha\right) \\ &= \delta f(\overline{\alpha}_j)\rho_{-j}\left[\left(\frac{1}{\rho_i}\right)\left(\frac{c}{\tau_i} - \frac{\delta}{1-\delta} b + r + \int_{\alpha_L}^{\overline{\alpha}_i} \alpha f(\alpha)\, d\alpha\right) - \overline{\alpha}_i\right] \\ &= \delta f(\overline{\alpha}_j)\frac{\rho}{\rho_i\rho_j}\left(\frac{c}{\tau_i} - \frac{\delta}{1-\delta} b + r + \int_{\alpha_L}^{\overline{\alpha}_i} \alpha f(\alpha)\, d\alpha - \overline{\alpha}_i\rho_i\right) \end{aligned}$$

Note that: $\lim_{\delta\to 0} \Psi^i_{\overline{\alpha}_j} = 0$.

Other useful derivatives are:

$$\frac{\partial\Psi^i}{\partial\tau_i} = (1-\delta\rho_{-i})\frac{c}{\tau_i^2} \quad \text{and} \quad \frac{\partial\Psi^i}{\partial r} = \delta\rho_{-i} \quad \text{and} \quad \frac{\partial\Psi^i}{\partial\tau_j} = 0$$

Define the Jacobian matrix as:

$$\mathbf{J} = \begin{bmatrix} \Psi^1_{\overline{\alpha}_1} & \ldots & \Psi^1_{\overline{\alpha}_n} \\ \ldots & \ldots & \ldots \\ \Psi^n_{\overline{\alpha}_1} & \ldots & \Psi^n_{\overline{\alpha}_n} \end{bmatrix}$$

Note that each entry in matrix $\mathbf{J}$ approaches either 0 or 1 in the limit as δ approaches 0. So in the limit, matrix $\mathbf{J}$ approaches the identity matrix, $\mathbf{I}$:

$$\lim_{\delta \to 0} \mathbf{J} = \begin{bmatrix} 1 & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & 1 \end{bmatrix} = \mathbf{I}$$

So the Jacobian matrix is nonsingular in the limit because $\det(\mathbf{I}) = 1 \neq 0$. This ensures that we can derive comparative statics using the implicit function theorem.

Additionally, we can prove the following intermediate result:

Lemma 1. *In equilibrium,* $\overline{\alpha}_i < \frac{c}{\tau_i}$.

Proof of Lemma 1. By the derivations above, $\Psi^i_{\overline{\alpha}_i} > 0$. Note that:

$$\Psi^i \left(\overline{\alpha}_i = \frac{c}{\tau_i} \right) = \delta \rho_{-i} (1 - \rho_i) \frac{c}{\tau_i} + (1 - \delta \rho_{-i}) \frac{\delta}{1 - \delta} b + \delta \rho_{-i} \left(r + \int_{\alpha_L}^{\frac{c}{\tau_i}} \alpha f(\alpha) \, d\alpha \right) > 0$$

So the equilibrium value of $\overline{\alpha}_i$ that solves $\Psi^i(\overline{\alpha}_i) = 0$ must be less than $\frac{c}{\tau_i}$. □

General Comparative Statics

Proposition 2: When its own trade stake (τ_i) increases, player i is more likely to file in any given period.

Proof of Proposition 2. Because the indexing of players is arbitrary, we solve for the impact of τ_1 on $\overline{\alpha}_1$. By the implicit function theorem:

$$\frac{\partial \overline{\alpha}_1}{\partial \tau_1} = \frac{-\det(\mathbf{B})}{\det(\mathbf{J})} \quad \text{where} \quad \mathbf{B} = \begin{bmatrix} \Psi^1_{\tau_1} & \Psi^1_{\overline{\alpha}_2} & \dots & \Psi^1_{\overline{\alpha}_n} \\ \Psi^2_{\tau_1} & \Psi^2_{\overline{\alpha}_2} & \dots & \Psi^2_{\overline{\alpha}_n} \\ \dots & \dots & \dots & \dots \\ \Psi^n_{\tau_1} & \Psi^n_{\overline{\alpha}_2} & \dots & \Psi^n_{\overline{\alpha}_n} \end{bmatrix}$$

Then the transpose of $\mathbf{B}$, which is denoted by $\mathbf{B^T}$, is:

$$\mathbf{B^T} = \begin{bmatrix} \Psi^1_{\tau_1} & 0 & \dots & 0 \\ \Psi^1_{\overline{\alpha}_2} & \Psi^2_{\overline{\alpha}_2} & \dots & \Psi^n_{\overline{\alpha}_2} \\ \dots & \dots & \dots & \dots \\ \Psi^1_{\overline{\alpha}_n} & \Psi^2_{\overline{\alpha}_n} & \dots & \Psi^n_{\overline{\alpha}_n} \end{bmatrix} \quad \text{because } \Psi^j_{\tau_1} = 0 \text{ for } j = 2, \dots, n$$

Let $\mathbf{B^T_{ij}}$ denote the submatrix formed by deleting the i-th row and j-th column of matrix $\mathbf{B^T}$. Then:

$$\det(\mathbf{B}) = \det\left(\mathbf{B^T}\right) = \Psi^1_{\tau_1} \det\left(\mathbf{B^T_{11}}\right) = (1 - \delta \rho_{-1}) \frac{c}{\tau_1^2} \det\left(\mathbf{B^T_{11}}\right)$$

By the argument above regarding the Jacobian matrix, $\lim_{\delta \to 0} \mathbf{B^T_{11}} = \mathbf{I}$. So:

$$\lim_{\delta \to 0} \det(\mathbf{B}) = \frac{c}{\tau_1^2} \det(\mathbf{I}) = \frac{c}{\tau_1^2} > 0 \quad \text{which implies} \quad \frac{\partial \overline{\alpha}_1}{\partial \tau_1} < 0 \;\; \text{for small } \delta$$

A lower value of $\overline{\alpha}_i$ means that player i is more likely to file (be type $\alpha_{it} \geq \overline{\alpha}_i$). □

Proposition 3: When another player's trade stake (τ_j) increases, player i is less likely to file in any given period.

Proof of Proposition 3. Because the indexing of players is arbitrary, we solve for the impact of τ_n on $\overline{\alpha}_1$. By the implicit function theorem:

$$\frac{\partial \overline{\alpha}_1}{\partial \tau_n} = \frac{-\det(\mathbf{C})}{\det(\mathbf{J})} \quad \text{where} \quad \mathbf{C} = \begin{bmatrix} \Psi^1_{\tau_n} & \Psi^1_{\overline{\alpha}_2} & \dots & \Psi^1_{\overline{\alpha}_{n-1}} & \Psi^1_{\overline{\alpha}_n} \\ \Psi^2_{\tau_n} & \Psi^2_{\overline{\alpha}_2} & \dots & \Psi^2_{\overline{\alpha}_{n-1}} & \Psi^2_{\overline{\alpha}_n} \\ \dots & \dots & \dots & \dots & \dots \\ \Psi^{n-1}_{\tau_n} & \Psi^{n-1}_{\overline{\alpha}_2} & \dots & \Psi^{n-1}_{\overline{\alpha}_{n-1}} & \Psi^{n-1}_{\overline{\alpha}_n} \\ \Psi^n_{\tau_n} & \Psi^n_{\overline{\alpha}_2} & \dots & \Psi^n_{\overline{\alpha}_{n-1}} & \Psi^n_{\overline{\alpha}_n} \end{bmatrix}$$

Then the transpose of $\mathbf{C}$, which is denoted by $\mathbf{C^T}$, is:

$$\mathbf{C^T} = \begin{bmatrix} 0 & 0 & \dots & 0 & \Psi^n_{\tau_n} \\ \Psi^1_{\overline{\alpha}_2} & \Psi^2_{\overline{\alpha}_2} & \dots & \Psi^{n-1}_{\overline{\alpha}_2} & \Psi^n_{\overline{\alpha}_2} \\ \dots & \dots & \dots & \dots & \dots \\ \Psi^1_{\overline{\alpha}_{n-1}} & \Psi^2_{\overline{\alpha}_{n-1}} & \dots & \Psi^{n-1}_{\overline{\alpha}_{n-1}} & \Psi^n_{\overline{\alpha}_{n-1}} \\ \Psi^1_{\overline{\alpha}_n} & \Psi^2_{\overline{\alpha}_n} & \dots & \Psi^{n-1}_{\overline{\alpha}_n} & \Psi^n_{\overline{\alpha}_n} \end{bmatrix} \quad \text{because} \quad \Psi^j_{\tau_n} = 0 \text{ for } j = 1, \dots, n-1$$

We can define a new matrix $\mathbf{D}$ by rearranging matrix $\mathbf{C^T}$ using $(n-1)$ column switches, and $(n-2)$ row switches:

$$\mathbf{C^T} \quad \text{after } (n-1) \text{ column switches is} \quad \begin{bmatrix} \Psi^n_{\tau_n} & 0 & 0 & \dots & 0 \\ \Psi^n_{\overline{\alpha}_2} & \Psi^1_{\overline{\alpha}_2} & \Psi^2_{\overline{\alpha}_2} & \dots & \Psi^{n-1}_{\overline{\alpha}_2} \\ \dots & \dots & \dots & \dots & \dots \\ \Psi^n_{\overline{\alpha}_{n-1}} & \Psi^1_{\overline{\alpha}_{n-1}} & \Psi^2_{\overline{\alpha}_{n-1}} & \dots & \Psi^{n-1}_{\overline{\alpha}_{n-1}} \\ \Psi^n_{\overline{\alpha}_n} & \Psi^1_{\overline{\alpha}_n} & \Psi^2_{\overline{\alpha}_n} & \dots & \Psi^{n-1}_{\overline{\alpha}_n} \end{bmatrix}$$

$$\text{and after } (n-2) \text{ row switches is} \quad \begin{bmatrix} \Psi^n_{\tau_n} & 0 & 0 & \dots & 0 \\ \Psi^n_{\overline{\alpha}_n} & \Psi^1_{\overline{\alpha}_n} & \Psi^2_{\overline{\alpha}_n} & \dots & \Psi^{n-1}_{\overline{\alpha}_n} \\ \Psi^n_{\overline{\alpha}_2} & \Psi^1_{\overline{\alpha}_2} & \Psi^2_{\overline{\alpha}_2} & \dots & \Psi^{n-1}_{\overline{\alpha}_2} \\ \dots & \dots & \dots & \dots & \dots \\ \Psi^n_{\overline{\alpha}_{n-1}} & \Psi^1_{\overline{\alpha}_{n-1}} & \Psi^2_{\overline{\alpha}_{n-1}} & \dots & \Psi^{n-1}_{\overline{\alpha}_{n-1}} \end{bmatrix} \equiv \mathbf{D}$$

Note that:

$$\det(\mathbf{C}) = \det\left(\mathbf{C^T}\right) = (-1)^{2n-3}\det(\mathbf{D}) = -\Psi^n_{\tau_n}\det(\mathbf{D_{11}}) = -(1-\delta\rho_{-n})\frac{c}{\tau_n^2}\det(\mathbf{D_{11}})$$

To ascertain the sign of $\det(\mathbf{D_{11}})$, define the following matrix:

$$\mathbf{E} \equiv \mathbf{D_{11}} = \begin{bmatrix} \Psi^1_{\overline{\alpha}_n} & \Psi^2_{\overline{\alpha}_n} & \dots & \Psi^{n-1}_{\overline{\alpha}_n} \\ \Psi^1_{\overline{\alpha}_2} & \Psi^2_{\overline{\alpha}_2} & \dots & \Psi^{n-1}_{\overline{\alpha}_2} \\ \dots & \dots & \dots & \dots \\ \Psi^1_{\overline{\alpha}_{n-1}} & \Psi^2_{\overline{\alpha}_{n-1}} & \dots & \Psi^{n-1}_{\overline{\alpha}_{n-1}} \end{bmatrix}$$

$$\begin{aligned}
\text{So: } \det(\mathbf{E}) &= \sum_{k=1}^{n-1} (-1)^{k+1}\, \Psi^{k}_{\overline{\alpha}_n} \det(\mathbf{E_{1k}}) \\
&= \sum_{k=1}^{n-1} (-1)^{k+1} \left[\delta f(\overline{\alpha}_n) \frac{\rho}{\rho_k \rho_n} \left(\frac{c}{\tau_k} - \frac{\delta}{1-\delta} b + r + \int_{\alpha_L}^{\overline{\alpha}_k} \alpha f(\alpha)\, d\alpha - \overline{\alpha}_k \rho_k \right) \right] \det(\mathbf{E_{1k}}) \\
&= \left[\frac{\delta f(\overline{\alpha}_n)\rho}{\rho_n} \right] \sum_{k=1}^{n-1} (-1)^{k+1} \left(\frac{1}{\rho_k}\right) \left(\frac{c}{\tau_k} - \frac{\delta}{1-\delta} b + r + \int_{\alpha_L}^{\overline{\alpha}_k} \alpha f(\alpha)\, d\alpha - \overline{\alpha}_k \rho_k \right) \det(\mathbf{E_{1k}})
\end{aligned}$$

So $\det(\mathbf{E}) > 0$ if and only if the following condition holds:

$$\phi \equiv \sum_{k=1}^{n-1} (-1)^{k+1} \left(\frac{1}{\rho_k}\right) \left(\frac{c}{\tau_k} - \frac{\delta}{1-\delta} b + r + \int_{\alpha_L}^{\overline{\alpha}_k} \alpha f(\alpha)\, d\alpha - \overline{\alpha}_k \rho_k \right) \det(\mathbf{E_{1k}}) > 0$$

By the argument above regarding the Jacobian matrix, $\lim_{\delta\to 0} \mathbf{E_{11}} = \mathbf{I}$. So $\lim_{\delta\to 0} \det(\mathbf{E_{11}}) = \det(\mathbf{I}) = 1$. For $k = 2, 3, \ldots, n-1$, calculating $\det(\mathbf{E_{1k}})$ requires that we remove the k-th column of $\mathbf{E}$. This removes $\Psi^{k}_{\overline{\alpha}_k}$ from the k-th row of $\mathbf{E}$. Since all other entries in the k-th row of $\mathbf{E}$ approach 0 as δ approaches 0, $\lim_{\delta\to 0} \det(\mathbf{E_{1k}}) = 0$. So:

$$\lim_{\delta\to 0} \phi = \left(\frac{1}{\rho_1}\right) \left(\frac{c}{\tau_1} - \overline{\alpha}_1 \rho_1 + r + \int_{\alpha_L}^{\overline{\alpha}_1} \alpha f(\alpha)\, d\alpha \right)$$

By Lemma 1, we know that $\overline{\alpha}_1 < \frac{c}{\tau_1}$. This implies that $0 < \frac{c}{\tau_1} - \overline{\alpha}_1 < \frac{c}{\tau_1} - \overline{\alpha}_1 \rho_1$. So $\lim_{\delta\to 0} \phi > 0$, which implies that $\det(\mathbf{E}) = \det(\mathbf{D_{11}}) > 0$. We can thus conclude that:

$$\det(\mathbf{C}) = -(1 - \delta \rho_{-n}) \frac{c}{\tau_n^2} \det(\mathbf{D_{11}}) < 0 \quad \Rightarrow \quad \frac{\partial \overline{\alpha}_1}{\partial \tau_n} > 0 \text{ for small } \delta$$

□

Proposition 4: When the legal merit of the case increases, each player is more likely to file the case in any given period.

Proof of Proposition 4. Because the indexing of players is arbitrary, we solve for the impact of r on $\overline{\alpha}_1$. By the implicit function theorem:

$$\frac{\partial \overline{\alpha}_1}{\partial r} = \frac{-\det(\mathbf{G})}{\det(\mathbf{J})} \text{ where } \mathbf{G} = \begin{bmatrix} \Psi^1_r & \Psi^1_{\overline{\alpha}_2} & \ldots & \Psi^1_{\overline{\alpha}_n} \\ \Psi^2_r & \Psi^2_{\overline{\alpha}_2} & \ldots & \Psi^2_{\overline{\alpha}_n} \\ \ldots & \ldots & \ldots & \ldots \\ \Psi^n_r & \Psi^n_{\overline{\alpha}_2} & \ldots & \Psi^n_{\overline{\alpha}_n} \end{bmatrix}$$

The transpose of $\mathbf{G}$, which is denoted by $\mathbf{G^T}$, is

$$\mathbf{G^T} = \begin{bmatrix} \Psi_r^1 & \Psi_r^2 & \dots & \Psi_r^n \\ \Psi_{\overline{\alpha}_2}^1 & \Psi_{\overline{\alpha}_2}^2 & \dots & \Psi_{\overline{\alpha}_2}^n \\ \dots & \dots & \dots & \dots \\ \Psi_{\overline{\alpha}_n}^1 & \Psi_{\overline{\alpha}_n}^2 & \dots & \Psi_{\overline{\alpha}_n}^n \end{bmatrix}$$

Note that:

$$\det(\mathbf{G}) = \det\left(\mathbf{G^T}\right) = \sum_{k=1}^{n} (-1)^{k+1}\, \Psi_r^k \det\left(\mathbf{G^T_{1k}}\right) = \sum_{k=1}^{n} (-1)^{k+1}\, \delta\rho_{-k} \det\left(\mathbf{G^T_{1k}}\right)$$

So $\det(\mathbf{G}) > 0$ if and only if the following condition holds:

$$\lambda \equiv \sum_{k=1}^{n} (-1)^{k+1}\, \rho_{-k} \det\left(\mathbf{G^T_{1k}}\right) > 0$$

By the argument above regarding the Jacobian matrix, $\lim_{\delta\to 0} \mathbf{G^T_{11}} = \mathbf{I}$. So $\lim_{\delta\to 0} \det(\mathbf{G^T_{11}}) = \det(\mathbf{I}) = 1$. For $k = 2, 3, \dots, n-1$, calculating $\det\left(\mathbf{G^T_{1k}}\right)$ requires that we remove the k-th column of $\mathbf{G^T}$. This removes $\Psi_{\overline{\alpha}_k}^k$ from the k-th row of $\mathbf{G^T}$. Since all other entries in the k-th row of $\mathbf{G^T}$ approach 0 as δ approaches 0, $\lim_{\delta\to 0} \det\left(\mathbf{G^T_{1k}}\right) = 0$. So:

$$\lim_{\delta\to 0} \lambda = \rho_{-1} > 0 \quad \Rightarrow \quad \lim_{\delta\to 0} \det(\mathbf{G}) > 0 \quad \Rightarrow \quad \frac{\partial\overline{\alpha}_1}{\partial r} < 0 \text{ for small } \delta$$

□

Diffusion Comparative Statics

Proposition 5: When the number of affected countries increases, each player is less likely to file in any given period.

Proof of Proposition 5. Suppose players have identical trade stakes, $\tau_i = \frac{\tau}{n}$. The logic from the proof of Proposition 1 ensures that for small $\delta > 0$, there exists a unique Bayesian Nash equilibrium in which each player has an interior cutpoint $\overline{\alpha}_i \in (\alpha_L, \alpha_H)$. Since our assumption that players have identical trade stakes makes the game symmetric, the system of $\Psi^i(\overline{\alpha})$-equations can be simplified to one equation, which I denote as Ψ^n, with one endogenous variable, $\overline{\alpha}_n$.

Let ρ_n denote the *ex ante* probability that an arbitrary player does not file the dispute when the game has n players. In equilibrium, each player in the n-player game uses cutpoint $\overline{\alpha}_n$, which is implicitly defined by:

$$\Psi^n = \overline{\alpha}_n\left[1 - \delta(\rho_n)^n\right] - \left[1 - \delta(\rho_n)^{n-1}\right]\left(\frac{cn}{\tau} - \frac{\delta}{1-\delta}b\right) + \delta(\rho_n)^{n-1}\left(r + \int_{\alpha_L}^{\overline{\alpha}_n} \alpha f(\alpha)\, d\alpha\right) = 0$$

Consider the value of function Ψ^n in the limit as δ becomes arbitrarily small:

$$\lim_{\delta\to 0} \Psi^n = \overline{\alpha}_n - \frac{cn}{\tau}$$

We can therefore identify the equilibrium cutpoint as δ becomes arbitrarily small:

$$\lim_{\delta\to 0} \Psi^n = 0 \quad \Leftrightarrow \quad \lim_{\delta\to 0} \overline{\alpha}_n = \frac{cn}{\tau}$$

By the same logic, the unique cutpoint for the $(n+1)$-player game, $\overline{\alpha}_{n+1}$, is implicitly defined by:

$$\begin{aligned}\Psi^{n+1} &= \overline{\alpha}_{n+1}\left[1-\delta\left(\rho_{n+1}\right)^{n+1}\right] - \left[1-\delta\left(\rho_{n+1}\right)^{n}\right]\left[\frac{c\left(n+1\right)}{\tau} - \frac{\delta}{1-\delta}b\right] \\ &\quad +\delta\left(\rho_{n+1}\right)^{n}\left(r + \int_{\alpha_L}^{\overline{\alpha}_{n+1}} \alpha f\left(\alpha\right) d\alpha\right) = 0\end{aligned}$$

and the following holds:

$$\lim_{\delta\to 0} \Psi^{n+1} = \overline{\alpha}_{n+1} - \frac{c\left(n+1\right)}{\tau} = 0 \quad \Leftrightarrow \quad \lim_{\delta\to 0} \overline{\alpha}_{n+1} = \frac{c\left(n+1\right)}{\tau}$$

So $\lim_{\delta\to 0} \overline{\alpha}_n < \lim_{\delta\to 0} \overline{\alpha}_{n+1}$. This means that each player is less likely to file when the number of players increases and δ is small. □

Proposition 6: When condition (6) holds and the number of players increases, the overall probability that the case is filed by at least one player decreases.

Proof of Proposition 6. Suppose players have identical trade stakes, $\tau_i = \frac{\tau}{n}$. Conditional on reaching period t, the probability that at least one state files the case in period t when there are n players is: $1-(\rho_n)^n$. This probability is decreasing in n iff: $(\rho_n)^n < (\rho_{n+1})^{n+1}$. By the derivations in the Proof of Proposition 5:

$$\lim_{\delta\to 0} \left(\rho_n\right)^n = F\left(\frac{cn}{\tau}\right)^n \quad \text{and} \quad \lim_{\delta\to 0} \left(\rho_{n+1}\right)^{n+1} = F\left(\frac{c\left(n+1\right)}{\tau}\right)^{n+1}$$

So for small $\delta > 0$, Proposition 6 holds for probability distributions and parameters such that:

$$F\left(\frac{cn}{\tau}\right)^n < F\left(\frac{c\left(n+1\right)}{\tau}\right)^{n+1} \tag{6}$$

□

Proposition 7: In observable WTO disputes, cases that challenge more diffuse policies will, on average, have more enforcement delay when condition (6) holds.

Proof of Proposition 7. Suppose players have identical trade stakes, $\tau_i = \frac{\tau}{n}$. For any period t, we can model the probability that the case is filed by at least one player as a binomial random variable. Suppose there is a "failure" if no one files the case, and a "success" if at least one player files the case. Then the probability of a failure is $(\rho_n)^n$ and the probability of a success is $1-(\rho_n)^n$.

Let X denote the number of time periods until the first success. Then X is a geometric random variable and:

$$\Pr\left(X=t\right) = \left[\left(\rho_n\right)^n\right]^{t-1}\left[1-\left(\rho_n\right)^n\right]$$

The expected number of time periods until a success in the n-player game is:

$$E[X|n] = \sum_{t=1}^{\infty} t\left[(\rho_n)^n\right]^{t-1}\left[1-(\rho_n)^n\right] = \frac{1}{1-(\rho_n)^n}$$

To identify the impact of n, note that:

$$E[X|n] < E[X|n+1] \quad \Leftrightarrow \quad \frac{1}{1-(\rho_n)^n} < \frac{1}{1-(\rho_{n+1})^{(n+1)}} \quad \Leftrightarrow \quad (\rho_n)^n < (\rho_{n+1})^{n+1}$$

This holds whenever condition (6) holds. □

Proposition 8: In observable WTO disputes, cases that challenge diffuse policies will, on average, have more legal merit than cases that challenge concentrated policies.

Proof of Proposition 8. Suppose players have identical trade stakes, $\tau_i = \frac{\tau}{n}$. By the Proof of Proposition 1, the marginal benefit for player i of type α_i from filing the case when there are n players is:

$$\Psi^n(\alpha_i) \quad = \quad \alpha_i\left[1-\delta(\rho_n)^n\right] - \left[1-\delta(\rho_n)^{n-1}\right]\left(\frac{cn}{\tau} - \frac{\delta}{1-\delta}b\right) + \delta(\rho_n)^{n-1}\left(r + \int_{\alpha_L}^{\alpha_i} xf(x)\,dx\right)$$

So:

$$\Psi^n_r(\alpha_i) = \delta(\rho_n)^{n-1} > 0 \quad \text{and} \quad \lim_{r\to\infty}\Psi^n(\alpha_i) = \infty > 0$$
$$\text{and} \quad \lim_{\delta\to 0}\Psi^n(\alpha_i|r=0) = \alpha_i - \frac{cn}{\tau} < 0 \Leftrightarrow \alpha_i < \frac{cn}{\tau}$$

So for small δ and very high α_i-values, player i always files the case, regardless of the value of r. But for small δ and low α_i-values, the intermediate value theorem ensures that there exists a unique critical value $\overline{r}(\alpha_i, n) > 0$ such that $\Psi^n(\alpha_i|\overline{r}(\alpha_i, n)) = 0$, meaning that a player of type α_i files if and only if $\overline{r}(\alpha_i, n) \leq r$.

Also note that the marginal benefit for player i of type α_i from filing the case when there are $n+1$ players is:

$$\Psi^{n+1}(\alpha_i) \quad = \quad \alpha_i\left[1-\delta(\rho_{n+1})^{n+1}\right] - \left[1-\delta(\rho_{n+1})^n\right]\left(\frac{c(n+1)}{\tau} - \frac{\delta}{1-\delta}b\right)$$
$$+\,\delta(\rho_{n+1})^n\left(r + \int_{\alpha_L}^{\alpha_i} xf(x)\,dx\right)$$

Given the logic above, for small δ and low α_i-values there exists a unique critical value for the $(n+1)$-player game, $\overline{r}(\alpha_i, n+1) > 0$, such that $\Psi^{n+1}(\alpha_i|\overline{r}(\alpha_i, n+1)) = 0$.

Finally, note that:

$$\lim_{\delta\to 0}\left[\Psi^n(\alpha_i) - \Psi^{n+1}(\alpha_i)\right] = \frac{c}{\tau} > 0$$

By definition of the critical value:

$$\Psi^n(\alpha_i|\overline{r}(\alpha_i,n)) = 0 = \Psi^{n+1}(\alpha_i|\overline{r}(\alpha_i,n+1)) < \Psi^n(\alpha_i|\overline{r}(\alpha_i,n+1)) \quad \text{for small } \delta$$

This implies that $\overline{r}(\alpha_i,n) < \overline{r}(\alpha_i,n+1)$. So as the number of players increases, larger values of r will be necessary for a player of type $\alpha_i < \frac{cn}{\tau}$ to file the case. □

A.2 Robustness: Increase in Players While Trade Stakes Held Constant

Claim 1: Each individual in an n-player game is less likely to file if a new individual becomes affected by the trade policy.

Proof of Claim 1. Suppose we have an n-player game with trade stakes, $\tau^n \equiv (\tau_1^n, \tau_2^n, \ldots, \tau_n^n)$. By the proof of proposition 1, equilibrium behavior is defined by a vector, $\overline{\alpha}^n \equiv (\overline{\alpha}_1^n, \overline{\alpha}_2^n, \ldots, \overline{\alpha}_n^n)$, which is in turn defined by a system of equations:

$$\begin{array}{rcl} \Psi_1^n(\overline{\alpha}^n) & = & 0 \\ \Psi_2^n(\overline{\alpha}^n) & = & 0 \\ \cdots & \cdots & \cdots \\ \Psi_n^n(\overline{\alpha}^n) & = & 0 \end{array}$$

where:

$$\begin{aligned} \Psi_i^n(x,\tau^n) &\equiv x\left[1-\delta F(x)\prod_{j\neq i}F(\overline{\alpha}_j^n)\right] - \left[1-\delta\prod_{j\neq i}F(\overline{\alpha}_j^n)\right]\left(\frac{c}{\tau_i^n}-\frac{\delta}{1-\delta}b\right) \\ &\quad +\delta\prod_{j\neq i}F(\overline{\alpha}_j^n)\left(r+\int_{\alpha_L}^{x}\alpha f(\alpha)\,d\alpha\right) \\ &= x-\frac{c}{\tau_i^n}+\frac{\delta}{1-\delta}b+\delta\prod_{j\neq i}F(\overline{\alpha}_j^n)\left[\frac{c}{\tau_i^n}-\frac{\delta}{1-\delta}b+r+\int_{\alpha_L}^{x}(\alpha-x)f(\alpha)\,d\alpha\right] \end{aligned}$$

Now consider an $n+1$ player game where $\tau_i^{n+1} = \tau_i^n$ for all $i = 1, 2, \ldots, n$ and $\tau_{n+1}^{n+1} > 0$. Again, equilibrium behavior is defined by a vector, $\overline{\alpha}^{n+1} \equiv (\overline{\alpha}_1^{n+1}, \overline{\alpha}_2^{n+1}, \ldots, \overline{\alpha}_n^{n+1})$, which is in turn defined by a system of equations:

$$\begin{array}{rcl} \Psi_1^{n+1}(\overline{\alpha}^{n+1}) & = & 0 \\ \Psi_2^{n+1}(\overline{\alpha}^{n+1}) & = & 0 \\ \cdots & \cdots & \cdots \\ \Psi_{n+1}^{n+1}(\overline{\alpha}^{n+1}) & = & 0 \end{array}$$

where:

$$\Psi_i^{n+1}\left(x,\tau^{n+1}\right) \equiv x\left[1-\delta F(x)\prod_{j\neq i}F\left(\overline{\alpha}_j^{n+1}\right)\right]-\left[1-\delta\prod_{j\neq i}F\left(\overline{\alpha}_j^{n+1}\right)\right]\left(\frac{c}{\tau_i^{n+1}}-\frac{\delta}{1-\delta}b\right)$$

$$+\delta\prod_{j\neq i}F\left(\overline{\alpha}_j^{n+1}\right)\left(r+\int_{\alpha_L}^{x}\alpha f(\alpha)\,d\alpha\right)$$

$$= x-\frac{c}{\tau_i^{n+1}}+\frac{\delta}{1-\delta}b+\delta\prod_{j\neq i}F\left(\overline{\alpha}_j^{n+1}\right)\left[\frac{c}{\tau_i^{n+1}}-\frac{\delta}{1-\delta}b+r+\int_{\alpha_L}^{x}(\alpha-x)f(\alpha)\,d\alpha\right]$$

Define the difference function for each $i=1,2,\ldots,n$:

$$\Delta_i^n(x)\equiv\Psi_i^n(x,\tau^n)-\Psi_i^{n+1}\left(x,\tau^{n+1}\right)$$

Then:

$$\Delta_i^n(x)=x-\frac{c}{\tau_i^n}+\frac{\delta}{1-\delta}b+\delta\prod_{j\neq i}F\left(\overline{\alpha}_j^n\right)\left[\frac{c}{\tau_i^n}-\frac{\delta}{1-\delta}b+r+\int_{\alpha_L}^{x}(\alpha-x)f(\alpha)\,d\alpha\right]$$

$$-\left\{x-\frac{c}{\tau_i^{n+1}}+\frac{\delta}{1-\delta}b+\delta\prod_{j\neq i}F\left(\overline{\alpha}_j^{n+1}\right)\left[\frac{c}{\tau_i^{n+1}}-\frac{\delta}{1-\delta}b+r+\int_{\alpha_L}^{x}(\alpha-x)f(\alpha)\,d\alpha\right]\right\}$$

Because $\tau_i^{n+1}=\tau_i^n$ for all $i=1,2,\ldots,n$:

$$\Delta_i^n(x)=\delta\left[\prod_{j\neq i}F\left(\overline{\alpha}_j^n\right)-\prod_{j\neq i}F\left(\overline{\alpha}_j^{n+1}\right)\right]\left[\frac{c}{\tau_i^n}-\frac{\delta}{1-\delta}b+r+\int_{\alpha_L}^{x}(\alpha-x)f(\alpha)\,d\alpha\right]$$

Case 1: If $\prod_{j\neq i}F\left(\overline{\alpha}_j^n\right)<\prod_{j\neq i}F\left(\overline{\alpha}_j^{n+1}\right)$, then $\Delta_i^n(x)<0$, meaning that $\Psi_i^n(x)<\Psi_i^{n+1}(x)$. This in turn would imply that $\overline{\alpha}_i^{n+1}<\overline{\alpha}_i^n$. This holds for any arbitrary player $i=1,2,\ldots,n$. So it must be true that for any $i=1,2,\ldots,n$:

$$\prod_{j\neq i}F\left(\overline{\alpha}_j^{n+1}\right)=F\left(\overline{\alpha}_{n+1}^{n+1}\right)\prod_{j\neq i\wedge j\in\{1,2,\ldots n\}}F\left(\overline{\alpha}_j^{n+1}\right)<\prod_{j\neq i\wedge j\in\{1,2,\ldots n\}}F\left(\overline{\alpha}_j^{n+1}\right)<\prod_{j\neq i\wedge j\in\{1,2,\ldots n\}}F\left(\overline{\alpha}_j^n\right)$$

This creates a contradiction.

Case 2: If $\prod_{j\neq i}F\left(\overline{\alpha}_j^n\right)=\prod_{j\neq i}F\left(\overline{\alpha}_j^{n+1}\right)$, then $\Delta_i^n(x)=0$, meaning that $\Psi_i^n(x)=\Psi_i^{n+1}(x)$. This in turn would imply that $\overline{\alpha}_i^{n+1}=\overline{\alpha}_i^n$. So for any $i=1,2,\ldots,n$:

$$\prod_{j\neq i}F\left(\overline{\alpha}_j^{n+1}\right)=F\left(\overline{\alpha}_{n+1}^{n+1}\right)\prod_{j\neq i\wedge j\in\{1,2,\ldots n\}}F\left(\overline{\alpha}_j^{n+1}\right)<\prod_{j\neq i\wedge j\in\{1,2,\ldots n\}}F\left(\overline{\alpha}_j^{n+1}\right)=\prod_{j\neq i\wedge j\in\{1,2,\ldots n\}}F\left(\overline{\alpha}_j^n\right)$$

Once again, this creates a contradiction.

Case 3: So it must be true that $\prod_{j\neq i}F\left(\overline{\alpha}_j^n\right)>\prod_{j\neq i}F\left(\overline{\alpha}_j^{n+1}\right)$, meaning that $\Delta_i^n(x)>0$, and hence $\Psi_i^n(x) > \Psi_i^{n+1}(x)$, and hence $\overline{\alpha}_i^{n+1} > \overline{\alpha}_i^n$.

So each individual in the original set of n players is less likely to file when the set expands to include a new player $n+1$. □

Claim 2: For small δ, the overall likelihood of enforcement increases if a new individual becomes affected by the trade policy.

Proof of Claim 2. Define the probability that someone enforces in the n-player game as:

$$\beta^n = 1 - \prod_{k=1}^{n} F\left(\overline{\alpha}_k^n\right)$$

Define the difference function:

$$\Gamma^n \equiv \beta^{n+1} - \beta^n = \prod_{k=1}^{n} F\left(\overline{\alpha}_k^n\right) - \prod_{k=1}^{n+1} F\left(\overline{\alpha}_k^{n+1}\right)$$

Note that:

$$\begin{aligned}
\lim_{\delta\to 0} \Psi_i^n\left(x, \tau^n\right) &= x - \frac{c}{\tau_i^n} = 0 \quad \Leftrightarrow \quad \lim_{\delta\to 0} \overline{\alpha}_i^n = \frac{c}{\tau_i^n} \\
\lim_{\delta\to 0} \Psi_i^{n+1}\left(x, \tau^{n+1}\right) &= x - \frac{c}{\tau_i^{n+1}} = 0 \quad \Leftrightarrow \quad \lim_{\delta\to 0} \overline{\alpha}_i^{n+1} = \frac{c}{\tau_i^{n+1}}
\end{aligned}$$

So:

$$\lim_{\delta\to 0} \overline{\alpha}_i^n = \lim_{\delta\to 0} \overline{\alpha}_i^{n+1} \quad \text{for any } i = 1, 2, \ldots, n \text{ because } \tau_i^n = \tau_i^{n+1}$$

So:

$$\lim_{\delta\to 0} \Gamma^n = \prod_{k=1}^{n} F\left(\frac{c}{\tau_k^n}\right) - \prod_{k=1}^{n+1} F\left(\frac{c}{\tau_k^{n+1}}\right) = \prod_{k=1}^{n} F\left(\frac{c}{\tau_k^n}\right)\left[1 - F\left(\frac{c}{\tau_{n+1}^{n+1}}\right)\right] > 0$$

So for small δ, increasing the number of players increases the overall likelihood of enforcement. □

A.3 Robustness: Common Reward as a Function of Diffuseness, $r(n)$

Let the common reward of litigation be a function of diffuseness, $r(n)$.
Recall that for an n-player game, the equilibrium cutpoint, x, is defined by:

$$\Psi^n(x) \equiv x\left[1 - \delta F(x)^n\right] - \left[1 - \delta F(x)^{n-1}\right]\left(\frac{cn}{\tau} - \frac{\delta}{1-\delta} b\right) + \delta F(x)^{n-1}\left[r(n) + \int_{\alpha_L}^{x} \alpha f(\alpha)\, d\alpha\right] = 0$$

And

$$\frac{\partial \Psi^n(x)}{\partial x} > 0$$

A player is less likely to file as diffuseness (n) increases iff: $\overline{\alpha}_n < \overline{\alpha}_{n+1}$.
Define the difference function:

$$\Delta(x) \equiv \Psi^n(x) - \Psi^{n+1}(x)$$

Our results hold generally iff:

$$\Delta(x) > 0 \qquad \text{for: } x = \overline{\alpha}_n$$

Note that:

$$\begin{aligned}
\Delta(x) &= x\left[1-\delta F(x)^n\right] - \left[1-\delta F(x)^{n-1}\right]\left(\frac{cn}{\tau} - \frac{\delta}{1-\delta}b\right) + \delta F(x)^{n-1}\left[r(n) + \int_{\alpha_L}^{x} \alpha f(\alpha)\,d\alpha\right] \\
&\quad - \left\{x\left[1-\delta F(x)^{n+1}\right] - \left[1-\delta F(x)^n\right]\left(\frac{c(n+1)}{\tau} - \frac{\delta}{1-\delta}b\right) + \delta F(x)^n\left[r(n+1) + \int_{\alpha_L}^{x} \alpha f(\alpha)\,d\alpha\right]\right\} \\
&= \frac{c}{\tau} - \delta x\left[1-F(x)\right]F(x)^n + \delta\left[1-F(x)\right]F(x)^{n-1}\left[\int_{\alpha_L}^{x} \alpha f(\alpha)\,d\alpha - \frac{\delta}{1-\delta}b\right] \\
&\quad + \delta F(x)^{n-1}\left\{\frac{cn}{\tau} + r(n) - F(x)\left[\frac{c(n+1)}{\tau} + r(n+1)\right]\right\} \\
&= \frac{c}{\tau} + \delta\left[1-F(x)\right]F(x)^{n-1}\left[\int_{\alpha_L}^{x} \alpha f(\alpha)\,d\alpha - \frac{\delta}{1-\delta}b - xF(x)\right] \\
&\quad + \delta F(x)^{n-1}\left\{\left[1-F(x)\right]\frac{cn}{\tau} - F(x)\frac{c}{\tau} + r(n) - F(x)r(n+1)\right\} \\
&= \left[1-\delta F(x)^n\right]\frac{c}{\tau} - \delta\left[1-F(x)\right]F(x)^{n-1}\left[\int_{\alpha_L}^{x} (x-\alpha) f(\alpha)\,d\alpha + \frac{\delta}{1-\delta}b\right] \\
&\quad + \delta F(x)^{n-1}\left\{\left[1-F(x)\right]\frac{cn}{\tau} + r(n) - F(x)r(n+1)\right\}
\end{aligned}$$

So $\Delta(x) > 0$ iff:

$$\begin{aligned}
&\delta\left[1-F(x)\right]F(x)^{n-1}\left[\int_{\alpha_L}^{x} (x-\alpha) f(\alpha)\,d\alpha + \frac{\delta}{1-\delta}b\right] \\
&\qquad < \left[1-\delta F(x)^n\right]\frac{c}{\tau} + \delta F(x)^{n-1}\left\{\left[1-F(x)\right]\frac{cn}{\tau} + r(n) - F(x)r(n+1)\right\} \\
&\Leftrightarrow \delta\left[1-F(x)\right]\left[\int_{\alpha_L}^{x} (x-\alpha) f(\alpha)\,d\alpha + \frac{\delta}{1-\delta}b\right] \\
&\qquad < \left[\frac{1-\delta F(x)^n}{F(x)^{n-1}}\right]\frac{c}{\tau} + \delta\left[1-F(x)\right]\frac{cn}{\tau} + \delta\left[r(n) - F(x)r(n+1)\right]
\end{aligned}$$

- Case 1: If $r(n)$ is decreasing in n, then $r(n+1) < r(n)$. And it is always true that $F(x)r(n+1) < r(n+1)$. So $0 < r(n) - F(x)r(n+1)$, which implies that constraint is more easily satisfied.
- Case 2: If $r(n)$ is increasing in n, then $r(n) < r(n+1)$. If is unclear whether the term $r(n) - F(x)r(n+1)$ is positive or negative. This would depend on the shape of the distribution

function and other specific parameter values. But the necessary constraint for Proposition 5 to hold is more easily satisfied if δ or b are small, or c is large.

A.4 Robustness: Variation in Individual Litigation Cost

Let $c_i > 0$ denote player i's individual cost of litigation. Define the vector of litigation costs as $\overline{c} \equiv (c_1, c_2, \ldots, c_n)$. Suppose that $\frac{c_i}{\tau_i} \in (\alpha_L, \alpha_H)$ for every i.

Equilibrium Behavior

The logic for Proposition 1 continues to hold. The equilibrium strategies that are characterized by cutpoints $\overline{\alpha} = (\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n)$ are implicitly defined by the following system of n equations:

$$\begin{array}{rcl} \Psi^1 (\overline{\alpha}|\overline{c}) & = & 0 \\ \Psi^2 (\overline{\alpha}|\overline{c}) & = & 0 \\ \cdots & \cdots & \cdots \\ \Psi^n (\overline{\alpha}|\overline{c}) & = & 0 \end{array}$$

where:

$$\Psi^i \equiv \overline{\alpha}_i (1 - \delta\rho) - (1 - \delta\rho_{-i}) \left(\frac{c_i}{\tau_i} - \frac{\delta}{1-\delta} b \right) + \delta\rho_{-i} \left(r + \int_{\alpha_L}^{\overline{\alpha}_i} \alpha f(\alpha)\, d\alpha \right) \tag{7}$$

Intermediate Results and General Comparative Statics

All of the intermediate results continue to hold when the litigation cost is indexed, c_i. The proofs of Propositions 2-4 also continue to hold.

Diffusion Comparative Statics

Proposition 5: When the number of affected countries increases, each country is less likely to file in any given period.

Proof of Proposition 5. Suppose that players have identical trade stakes, $\tau_i = \frac{\tau}{n}$. The logic from the proof of Proposition 1 ensures that for small $\delta > 0$, there exists a unique Bayesian Nash equilibrium in which each player has an interior cutpoint $\overline{\alpha}_i \in (\alpha_L, \alpha_H)$. If different players have different litigation costs, then the system of n equations that define the $\overline{\alpha}_i$ values can no longer be simplified to a single equation (as in the main proofs).

Using equation (7), we can identify each player i's equilibrium cutpoint of the n-player game as δ becomes arbitrarily small:

$$\lim_{\delta \to 0} \Psi^i = 0 \quad \Leftrightarrow \quad \lim_{\delta \to 0} \overline{\alpha}_i = \frac{c_i n}{\tau}$$

Now hold the litigation costs for players 1 through n constant and add a player $n+1$ with cost parameter c_{n+1}. Define the new vector of litigation costs as $\overline{c}' \equiv (c_1, c_2, \ldots, c_n, c_{n+1})$. The newequilibrium cutpoints $\overline{\alpha}' = \left(\overline{\alpha}'_1, \overline{\alpha}'_2, \ldots, \overline{\alpha}'_n, \overline{\alpha}'_{n+1}\right)$ are implicitly defined by the following system of

$n + 1$ equations:

$$\begin{array}{rcl} \Gamma^1\left(\overline{\alpha}'|\overline{c}'\right) & = & 0 \\ \Gamma^2\left(\overline{\alpha}'|\overline{c}'\right) & = & 0 \\ \cdots & \cdots & \cdots \\ \Gamma^n\left(\overline{\alpha}'|\overline{c}'\right) & = & 0 \\ \Gamma^{n+1}\left(\overline{\alpha}'|\overline{c}'\right) & = & 0 \end{array}$$

where:

$$\Gamma^i \equiv \overline{\alpha}_i'\left(1-\delta\rho'\right) - \left(1-\delta\rho_{-i}'\right)\left(\frac{c_i(n+1)}{\tau} - \frac{\delta}{1-\delta}b\right) + \delta\rho_{-i}'\left(r + \int_{\alpha_L}^{\overline{\alpha}_i'} \alpha f\left(\alpha\right) d\alpha\right)$$

We can therefore identify the equilibrium cutpoint of the $(n+1)$-player game as δ becomes arbitrarily small:

$$\lim_{\delta\to 0} \Gamma^i = 0 \quad \Leftrightarrow \quad \lim_{\delta\to 0} \overline{\alpha}_i' = \frac{c_i(n+1)}{\tau}$$

So $\lim_{\delta\to 0} \overline{\alpha}_i < \lim_{\delta\to 0} \overline{\alpha}_i'$. This means that each player in the original n-player game is less likely to file when the number of players increases and δ is small. □

Proposition 6: When condition (8) holds and the number of affected players increases, the overall probability that the case is filed by at least one player decreases.

Proof of Proposition 6. Suppose players have identical trade stakes, $\tau_i = \frac{\tau}{n}$. Use the notation defined above in the Proof of Proposition 5. Conditional on reaching period t, the probability that at least one state files the case in period t when there are n players is:

$$1 - \prod_{i=1}^{n} \rho_i = 1 - \prod_{i=1}^{n} F\left(\overline{\alpha}_i\right)$$

Conditional on reaching period t, the probability that at least one state files the case in period t when there are $n+1$ players is:

$$1 - \prod_{i=1}^{n+1} \rho_i' = 1 - \prod_{i=1}^{n+1} F\left(\overline{\alpha}_i'\right)$$

So the probability that the case is filed in period t is decreasing in n iff:

$$\prod_{i=1}^{n} F\left(\overline{\alpha}_i\right) < \prod_{i=1}^{n+1} F\left(\overline{\alpha}_i'\right)$$

By the derivations in the Proof of Proposition 5:

$$\lim_{\delta\to 0} \prod_{i=1}^{n} F\left(\overline{\alpha}_i\right) = \prod_{i=1}^{n} F\left(\frac{c_i n}{\tau}\right) \quad \text{and} \quad \lim_{\delta\to 0} \prod_{i=1}^{n+1} F\left(\overline{\alpha}_i'\right) = \prod_{i=1}^{n+1} F\left(\frac{c_i(n+1)}{\tau_i}\right)$$

So for small $\delta > 0$, Proposition 6 holds for probability distributions and parameters such that:

$$\prod_{i=1}^{n} F\left(\frac{c_i n}{\tau}\right) < \prod_{i=1}^{n+1} F\left(\frac{c_i(n+1)}{\tau_i}\right) \tag{8}$$

□

Proposition 7: In observable WTO disputes, cases that challenge more diffuse policies will, on average, have more enforcement delay when condition (8) holds.

Proof of Proposition 7. Suppose players have identical trade stakes, $\tau_i = \frac{\tau}{n}$. For any period t, we can model the probability that the case is filed by at least one player as a binomial random variable. Suppose there is a "failure" if no one files the case, and a "success" if at least one player files the case. Then the probability of a failure is $\rho \equiv \prod_{i=1}^{n} \rho_i$ and the probability of a success is $1 - \rho$.

Let X denote the number of time periods until the first success. Then X is a geometric random variable and:

$$\Pr(X = t) = \rho^{t-1}(1-\rho)$$

The expected number of time periods until a success in the n-player game is:

$$E[X|n] = \sum_{t=1}^{\infty} t\rho^{t-1}(1-\rho) = \frac{1}{1-\rho}$$

Similarly, for the $(n+1)$-player game, denote the probability of a failure is $\rho' \equiv \prod_{i=1}^{n+1} \rho_i'$ and the probability of a success is $1 - \rho'$. Then the expected number of time periods until a success in the $(n+1)$-player game is:

$$E[X|n+1] = \sum_{t=1}^{\infty} t\left(\rho'\right)^{t-1}\left(1-\rho'\right) = \frac{1}{1-\rho'}$$

To identify the impact of n, note that:

$$E[X|n] < E[X|n+1] \quad \Leftrightarrow \quad \frac{1}{1-\rho} < \frac{1}{1-\rho'} \quad \Leftrightarrow \quad \rho < \rho'$$

Note that this is equivalent to condition (8). □

Proposition 8: In observable WTO disputes, cases that challenge diffuse policies will, on average, have more legal merit than cases that challenge concentrated policies.

Proof of Proposition 8. Suppose players have identical trade stakes, $\tau_i = \frac{\tau}{n}$. Use the notation defined above in the Proof of Proposition 5. In the n-player game with the litigation cost vector $\bar{c}$, the marginal benefit for player i of type α_i from filing the case is:

$$\Psi^i(\alpha_i|\bar{c}) = \alpha_i(1-\delta\rho) - (1-\delta\rho_{-i})\left(\frac{c_i n}{\tau} - \frac{\delta}{1-\delta}b\right) + \delta\rho_{-i}\left(r + \int_{\alpha_L}^{\alpha_i} xf(x)\,dx\right)$$

So:

$$\Psi_r^i(\alpha_i|\overline{c}) = \delta\rho_{-i} > 0 \quad \text{and} \quad \lim_{r\to\infty}\Psi^i(\alpha_i|\overline{c}) = \infty > 0$$
$$\text{and} \lim_{\delta\to 0}\Psi^i(\alpha_i|\overline{c}, r=0) = \alpha_i - \frac{c_i n}{\tau} < 0 \Leftrightarrow \alpha_i < \frac{c_i n}{\tau}$$

So for small δ and very high α_i-values, player i always files the case, regardless of the value of r. But for small δ and low α_i-values, the intermediate value theorem ensures that for each player i there exists a unique critical value $\overline{r}_i(\alpha_i, \overline{c}) > 0$ such that $\Psi^i(\alpha_i|\overline{c}, \overline{r}_i(\alpha_i, \overline{c})) = 0$, meaning that a player i of type α_i files if and only if $\overline{r}_i(\alpha_i, \overline{c}) \leq r$.

Also note that in the $(n+1)$-player game with the litigation cost vector $\overline{c}'$, the marginal benefit for player i of type α_i from filing the case is:

$$\Gamma^i\left(\alpha_i|\overline{c}'\right) = \alpha_i\left(1-\delta\rho'\right) - \left(1-\delta\rho'_{-i}\right)\left(\frac{c_i(n+1)}{\tau} - \frac{\delta}{1-\delta}b\right) + \delta\rho'_{-i}\left(r + \int_{\alpha_L}^{\alpha_i} xf(x)\,dx\right)$$

So:

$$\Gamma_r^i\left(\alpha_i|\overline{c}'\right) = \delta\rho'_{-i} > 0 \quad \text{and} \quad \lim_{r\to\infty}\Gamma^i\left(\alpha_i|\overline{c}'\right) = \infty > 0$$
$$\text{and} \lim_{\delta\to 0}\Gamma^i\left(\alpha_i|\overline{c}', r=0\right) = \alpha_i - \frac{c_i(n+1)}{\tau} < 0 \Leftrightarrow \alpha_i < \frac{c_i(n+1)}{\tau}$$

So for small δ and very high α_i-values, player i always files the case, regardless of the value of r. But for small δ and low α_i-values, the intermediate value theorem ensures that for each player i there exists a unique critical value $\overline{r}_i(\alpha_i, \overline{c}') > 0$ such that $\Gamma^i(\alpha_i|\overline{c}', \overline{r}_i(\alpha_i, \overline{c}')) = 0$, meaning that a player of type α_i files if and only if $\overline{r}_i(\alpha_i, \overline{c}') \leq r$.

Finally, note that:

$$\lim_{\delta\to 0}\left[\Psi^i(\alpha_i|\overline{c}) - \Gamma^i\left(\alpha_i|\overline{c}'\right)\right] = \frac{c_i}{\tau} > 0$$

By definition of the critical value:

$$\Psi^i(\alpha_i|\overline{c}, \overline{r}_i(\alpha_i, \overline{c})) = 0 = \Gamma^i\left(\alpha_i|\overline{c}', \overline{r}_i\left(\alpha_i, \overline{c}'\right)\right) < \Psi^i\left(\alpha_i|\overline{c}, \overline{r}_i\left(\alpha_i, \overline{c}'\right)\right) \quad \text{for small } \delta$$

This implies that $\overline{r}_i(\alpha_i, \overline{c}) < \overline{r}_i(\alpha_i, \overline{c}')$. So as the number of players increases, larger values of r will be necessary for player i of type $\alpha_i < \frac{c_i n}{\tau}$ to file the case. □

A.5 Robustness: Litigation Cost Decreasing in the Number of Players

Let c_n denote the cost of litigation in the n-player game. Assume that $c_{n+1} \leq c_n$ for all $n \in \mathbf{N}$. So increasing the number of players decreases the cost of litigation for each individual player.
For our diffusion results, we additionally assume that the following constraint holds:

$$c_n\left(\frac{n}{n+1}\right) < c_{n+1} \tag{9}$$

Equilibrium Behavior

If we replace the parameter c with c_n for the n-player game, then the logic for Proposition 1 continues to hold. Each player i's equilibrium cutpoint is defined by the following constraint:

$$\Psi^i \equiv \overline{\alpha}_i (1 - \delta\rho) - (1 - \delta\rho_{-i}) \left(\frac{c_n}{\tau_i} - \frac{\delta}{1-\delta} b \right) + \delta\rho_{-i} \left(r + \int_{\alpha_L}^{\overline{\alpha}_i} \alpha f(\alpha)\, d\alpha \right)$$

Intermediate Results and General Comparative Statics

All of the intermediate results continue to hold when the cost parameter is a function of the number of players, c_n. The proofs of Propositions 2-4 also continue to hold with the modified cost parameter.

Diffusion Comparative Statics

Proposition 5: When the number of affected countries increases and condition (9) holds, each country is less likely to file in any given period.

Proof of Proposition 5. Suppose players have identical trade stakes, $\tau_i = \frac{\tau}{n}$. The initial logic from the main proof of Proposition 5 still holds. The system of $\Psi^i(\overline{\alpha})$-equations can be simplified to one equation, which I denote as Ψ^n, with one endogenous variable, $\overline{\alpha}_n$.

Let ρ_n denote the *ex ante* probability that an arbitrary player does not file the dispute when the game has n players. In equilibrium, each player in the n-player game uses cutpoint $\overline{\alpha}_n$, which is implicitly defined by:

$$\Psi^n = \overline{\alpha}_n \left[1 - \delta(\rho_n)^n\right] - \left[1 - \delta(\rho_n)^{n-1}\right] \left(\frac{c_n n}{\tau} - \frac{\delta}{1-\delta} b \right) + \delta(\rho_n)^{n-1} \left(r + \int_{\alpha_L}^{\overline{\alpha}_n} \alpha f(\alpha)\, d\alpha \right) = 0$$

Consider the value of function Ψ^n in the limit as δ becomes arbitrarily small:

$$\lim_{\delta \to 0} \Psi^n = \overline{\alpha}_n - \frac{c_n n}{\tau}$$

We can therefore identify the equilibrium cutpoint as δ becomes arbitrarily small:

$$\lim_{\delta \to 0} \Psi^n = 0 \quad \Leftrightarrow \quad \lim_{\delta \to 0} \overline{\alpha}_n = \frac{c_n n}{\tau}$$

By the same logic, the unique cutpoint for the $(n+1)$-player game, $\overline{\alpha}_{n+1}$, is implicitly defined by:

$$\begin{aligned} \Psi^{n+1} \quad = \quad & \overline{\alpha}_{n+1} \left[1 - \delta(\rho_{n+1})^{n+1}\right] - \left[1 - \delta(\rho_{n+1})^n\right] \left(\frac{c_{n+1}}{\tau} - \frac{\delta}{1-\delta} b \right) \\ & + \delta(\rho_{n+1})^n \left(r + \int_{\alpha_L}^{\overline{\alpha}_{n+1}} \alpha f(\alpha)\, d\alpha \right) = 0 \end{aligned}$$

and the following holds:

$$\lim_{\delta \to 0} \Psi^{n+1} = \overline{\alpha}_{n+1} - \frac{c_{n+1}(n+1)}{\tau} = 0 \quad \Leftrightarrow \quad \lim_{\delta \to 0} \overline{\alpha}_{n+1} = \frac{c_{n+1}(n+1)}{\tau}$$

So:

$$\lim_{\delta \to 0} \overline{\alpha}_n < \lim_{\delta \to 0} \overline{\alpha}_{n+1} \quad \Leftrightarrow \quad c_n \left(\frac{n}{n+1} \right) < c_{n+1}$$

This means that each player is less likely to file when the number of players increases and δ is small and condition (9) holds. □

Proposition 6: When condition (10) holds and the number of players increases, the overall probability that the case is filed by at least one player decreases.

Proof of Proposition 6. Suppose players have identical trade stakes, $\tau_i = \frac{\tau}{n}$. Conditional on reaching period t, the probability that at least one state files the case in period t when there are n players is: $1 - (\rho_n)^n$. This probability is decreasing in n iff: $(\rho_n)^n < (\rho_{n+1})^{n+1}$. By the derivations in the Proof of Proposition 5:

$$\lim_{\delta \to 0} (\rho_n)^n = F\left(\frac{c_n n}{\tau}\right)^n \quad \text{and} \quad \lim_{\delta \to 0} (\rho_{n+1})^{n+1} = F\left(\frac{c_{n+1}(n+1)}{\tau}\right)^{n+1}$$

So for small $\delta > 0$, Proposition 6 holds for probability distributions and parameters such that:

$$F\left(\frac{c_n n}{\tau}\right)^n < F\left(\frac{c_{n+1}(n+1)}{\tau}\right)^{n+1} \tag{10}$$

□

Proposition 7: In observable WTO disputes, cases that challenge more diffuse policies will, on average, have more enforcement delay when condition (10) holds.

Proof of Proposition 7. Suppose players have identical trade stakes, $\tau_i = \frac{\tau}{n}$. For any period t, we can model the probability that the case is filed by at least one player as a binomial random variable. Suppose there is a "failure" if no one files the case, and a "success" if at least one player files the case. Then the probability of a failure is $(\rho_n)^n$ and the probability of a success is $1 - (\rho_n)^n$.

Let X denote the number of time periods until the first success. Then X is a geometric random variable and:

$$\Pr(X = t) = [(\rho_n)^n]^{t-1} [1 - (\rho_n)^n]$$

The expected number of time periods until a success in the n-player game is:

$$E[X|n] = \sum_{t=1}^{\infty} t [(\rho_n)^n]^{t-1} [1 - (\rho_n)^n] = \frac{1}{1 - (\rho_n)^n}$$

To identify the impact of n, note that:

$$E[X|n] < E[X|n+1] \quad \Leftrightarrow \quad \frac{1}{1 - (\rho_n)^n} < \frac{1}{1 - (\rho_{n+1})^{(n+1)}} \quad \Leftrightarrow \quad (\rho_n)^n < (\rho_{n+1})^{n+1}$$

This holds whenever condition (10) holds. □

Proposition 8: In observable WTO disputes, cases that challenge diffuse policies will, on average, have more legal merit than cases that challenge concentrated policies when condition (9) holds

Proof of Proposition 8. Suppose players have identical trade stakes, $\tau_i = \frac{\tau}{n}$. By the Proof of Proposition 1, the marginal benefit for player i of type α_i from filing the case when there are n players is:

$$\Psi^n(\alpha_i) = \alpha_i \left[1-\delta(\rho_n)^n\right] - \left[1-\delta(\rho_n)^{n-1}\right]\left(\frac{c_n n}{\tau} - \frac{\delta}{1-\delta}b\right) + \delta(\rho_n)^{n-1}\left(r + \int_{\alpha_L}^{\alpha_i} x f(x)\,dx\right)$$

So:

$$\Psi^n_r(\alpha_i) = \delta(\rho_n)^{n-1} > 0 \quad \text{and} \quad \lim_{r\to\infty}\Psi^n(\alpha_i) = \infty > 0$$
$$\text{and} \quad \lim_{\delta\to 0}\Psi^n(\alpha_i|r=0) = \alpha_i - \frac{c_n n}{\tau} < 0 \Leftrightarrow \alpha_i < \frac{c_n n}{\tau}$$

So for small δ and very high α_i-values, player i always files the case, regardless of the value of r. But for small δ and low α_i-values, the intermediate value theorem ensures that there exists a unique critical value $\overline{r}(\alpha_i, n) > 0$ such that $\Psi^n(\alpha_i|\overline{r}(\alpha_i, n)) = 0$, meaning that a player of type α_i files if and only if $\overline{r}(\alpha_i, n) \leq r$.

Also note that the marginal benefit for player i of type α_i from filing the case when there are $n+1$ players is:

$$\Psi^{n+1}(\alpha_i) = \alpha_i\left[1-\delta(\rho_{n+1})^{n+1}\right] - \left[1-\delta(\rho_{n+1})^n\right]\left(\frac{c_{n+1}(n+1)}{\tau} - \frac{\delta}{1-\delta}b\right) + \delta(\rho_{n+1})^n\left(r + \int_{\alpha_L}^{\alpha_i} x f(x)\,dx\right)$$

Given the logic above, for small δ and low α_i-values there exists a unique critical value for the $(n+1)$-player game, $\overline{r}(\alpha_i, n+1) > 0$, such that $\Psi^{n+1}(\alpha_i|\overline{r}(\alpha_i, n+1)) = 0$.

Finally, note that:

$$\lim_{\delta\to 0}\left[\Psi^n(\alpha_i) - \Psi^{n+1}(\alpha_i)\right] = \frac{c_{n+1}(n+1)}{\tau} - \frac{c_n n}{\tau} > 0 \quad \Leftrightarrow \quad c_n\left(\frac{n}{n+1}\right) < c_{n+1}$$

So when condition (9) holds, by definition of the critical value:

$$\Psi^n(\alpha_i|\overline{r}(\alpha_i, n)) = 0 = \Psi^{n+1}(\alpha_i|\overline{r}(\alpha_i, n+1)) < \Psi^n(\alpha_i|\overline{r}(\alpha_i, n+1)) \quad \text{for small } \delta$$

This implies that $\overline{r}(\alpha_i, n) < \overline{r}(\alpha_i, n+1)$. So as the number of players increases, larger values of r will be necessary for a player of type $\alpha_i < \frac{c_n n}{\tau}$ to file the case. □

A.6 Robustness: Panel Bias

To check the robustness of our results, we allow the public reward from litigation to be a function of the number of affected states, and assume it has the following functional form:

$$r_n \equiv \pi_n w + (1 - \pi_n)\, y$$

where $\pi_n \in [0, 1]$ is the probability of a pro-complainant ruling, w is the payoff if the complainant wins litigation, y is the payoff if the complainant does not win litigation, and $y < w$.

To isolate the effect of panel bias, we do not allow a player's individual trade shake, τ_i, to be a function of the number of affected states.

Equilibrium Behavior

The logic for Proposition 1 continues to hold. The equilibrium strategies that are characterized by cutpoints $\overline{\alpha} = (\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_n)$ are implicitly defined by the following system of n equations:

$$\begin{array}{rcl} \Psi^1(\overline{\alpha}) & = & 0 \\ \Psi^2(\overline{\alpha}) & = & 0 \\ \cdots & \cdots & \cdots \\ \Psi^n(\overline{\alpha}) & = & 0 \end{array}$$

where:

$$\Psi^i \equiv \overline{\alpha}_i (1 - \delta\rho) - (1 - \delta\rho_{-i}) \left(\frac{c}{\tau_i} - \frac{\delta}{1 - \delta} b \right) + \delta\rho_{-i} \left[\pi_n w_n + (1 - \pi_n)\, y_n \right] + \delta\rho_{-i} \int\limits_{\alpha_L}^{\overline{\alpha}_i} \alpha f(\alpha)\, d\alpha$$

Intermediate Results and General Comparative Statics

All of the intermediate results continue to hold when the reward payoff is indexed by the number of players, r_n. The proofs of Propositions 2-4 also continue to hold.

Impact of Panel Bias

Suppose that increasing the number of players increases the probability of a pro-complainant ruling (π_n), but does affect a player's individual trade shake, τ_i.

Proposition: When the number of players increases: (1) each player i more likely to file in any given period; and (2) the overall probability that the case is filed by at least one player increases.

Proof of Proposition. (1) Increasing the probability of a pro-complainant ruling (π_n) increases the expected public reward from filing the case, $r_n < r_{n+1}$. By the proof of Proposition 4, increasing the number of players therefore decreases each player's cutpoint: $\overline{\alpha}^i_{n+1} < \overline{\alpha}^i_n$. This means that each country more likely to file the case in any given period.

(2) Conditional on reaching period t, the probability that no state files the case in period t when there are n players is defined as:

$$\phi_n \equiv \prod_{i=1}^{n} F\left(\overline{\alpha}_n^i\right)$$

Then the probability that at least one state files the case in period t is: $1-\phi_n$. This probability is increasing in n iff: $\phi_{n+1} < \phi_n$. Note that:

$$\phi_{n+1} = \prod_{i=1}^{n+1} F\left(\overline{\alpha}_{n+1}^i\right) = F\left(\overline{\alpha}_{n+1}^{n+1}\right) \prod_{i=1}^{n} F\left(\overline{\alpha}_{n+1}^i\right) < \prod_{i=1}^{n} F\left(\overline{\alpha}_{n+1}^i\right) < \prod_{i=1}^{n} F\left(\overline{\alpha}_n^i\right) = \phi_n$$

□

Proposition: In observable WTO disputes, cases that challenge policies that affect more states will, on average, have less enforcement delay.

Proof of Proposition. As in the proof of Proposition 7, conditional on reaching period t of the game, we can model the probability that the case is filed by at least one player in period t as a binomial random variable. Suppose there is a "failure" if no player files the case, and a "success" if at least one player files the case. Then the probability of a failure is ϕ_n and the probability of a success is $1-\phi_n$.

Let X denote the number of time periods until the first success. Then X is a geometric random variable and:

$$\Pr\left(X=t\right) = \phi_n^{t-1}\left(1-\phi_n\right)$$

The expected number of time periods until a success in the n-player game is:

$$E\left[X|n\right] = \sum_{t=1}^{\infty} t\phi_n^{t-1}\left(1-\phi_n\right) = \frac{1}{1-\phi_n}$$

To identify the impact of n, note that:

$$E\left[X|n+1\right] < E\left[X|n\right] \quad \Leftrightarrow \quad \frac{1}{1-\phi_{n+1}} < \frac{1}{1-\phi_n} \quad \Leftrightarrow \quad \phi_{n+1} < \phi_n$$

This holds by the previous result. □

A.7 Robustness: Third Party Participation

Suppose that after the case is filed, affected states can join as a third party. This gives them access to possible private benefits while avoiding the cost of being the complainant. We assume that player i's ex ante expected benefit from possibly being a third party in the future (given that another player has filed the case) is $\sigma b\tau_i$ where $\sigma \in (0,1)$.

Given the notation defined above in section A.1 and conditional on reaching period t, player i's expected utility functions in period t are:

$$\begin{aligned} EU_{it}(\text{file}|\alpha_{it},\tau_i) &= \frac{\delta}{1-\delta}(r+b)\tau_i - c \\ EU_{it}(\text{don't file}|\alpha_{it},\tau_i) &= -\alpha_{it}\tau_i + (1-\rho_{-i})\frac{\delta}{1-\delta}(r+\sigma b)\tau_i + \rho_{-i}\delta V_i \end{aligned}$$

Equilibrium Behavior

Proof of Proposition 1. Player i has incentive to file iff:

$$\frac{\delta}{1-\delta}(r+b)\tau_i - c \geq -\alpha_{it}\tau_i + (1-\rho_{-i})\frac{\delta}{1-\delta}(r+\sigma b)\tau_i + \rho_{-i}\delta V_i$$

$$\Leftrightarrow \quad \alpha_{it} \geq \frac{c}{\tau_i} - \frac{\delta}{1-\delta}\left[1-\sigma(1-\rho_{-i})\right]b - \rho_{-i}\frac{\delta}{1-\delta}r + \frac{\delta\rho_{-i}}{\tau_i}V_i \equiv \overline{\alpha}_i \qquad (11)$$

Equilibrium behavior is therefore monotonic: high types ($\alpha_{it} > \overline{\alpha}_i$) will file, and low types ($\alpha_{it} < \overline{\alpha}_i$) will not file. So player i's best response function can be characterized by the value of $\overline{\alpha}_i$ defined in equation (11).

Let ρ_i denote the *ex ante* probability that player i does not file in a given time period. Then:

$$\rho_i = \Pr(\alpha_{it} < \overline{\alpha}_i) = F(\overline{\alpha}_i)$$

Let ρ denote the *ex ante* probability that no player files in a given time period, and ρ_{-i} denote the *ex ante* probability that no player besides i files in a given time period (per the description above). Then:

$$\rho = \prod_{k=1}^{n}\rho_k = \prod_{k=1}^{n}F(\overline{\alpha}_k) \quad \text{and} \quad \rho_{-i} = \prod_{j\neq i}\rho_j = \frac{\prod_{k=1}^{n}F(\overline{\alpha}_k)}{F(\overline{\alpha}_i)} = \frac{\rho}{\rho_i}$$

Also note that $\rho = \rho_i\rho_{-i}$.

In an interior equilibrium—an equilibrium in which $\overline{\alpha}_i \in [\alpha_L, \alpha_H]$ for all i—player i's continuation value is therefore:

$$\begin{aligned} V_i &= \int_{\alpha_L}^{\overline{\alpha}_i}\left[-\alpha\tau_i + (1-\rho_{-i})\frac{\delta}{1-\delta}(r+\sigma b)\tau_i + \rho_{-i}\delta V_i\right]f(\alpha)\,d\alpha \\ &\quad + \int_{\overline{\alpha}_i}^{\alpha_H}\left[\frac{\delta}{1-\delta}(r+b)\tau_i - c\right]f(\alpha)\,d\alpha \\ &= \rho_i\left[(1-\rho_{-i})\frac{\delta}{1-\delta}(r+\sigma b)\tau_i + \rho_{-i}\delta V_i\right] \\ &\quad + (1-\rho_i)\left[\frac{\delta}{1-\delta}(r+b)\tau_i - c\right] - \tau_i\int_{\alpha_L}^{\overline{\alpha}_i}\alpha f(\alpha)\,d\alpha \end{aligned} \qquad (12)$$

Manipulating equation (12) to isolate V_i yields:

$$V_i = \frac{1}{1-\delta\rho}\left[(1-\rho)\frac{\delta}{1-\delta}r\tau_i - (1-\rho_i)c + [1-(1-\sigma)\rho_i - \rho\sigma]\frac{\delta}{1-\delta}b\tau_i - \tau_i\int_{\alpha_L}^{\overline{\alpha}_i}\alpha f(\alpha)\,d\alpha\right] \quad (13)$$

Substituting equation (13) into equation (11) yields:

$$\overline{\alpha}_i = \frac{c}{\tau_i} - \frac{\delta}{1-\delta}[1-\sigma(1-\rho_{-i})]b - \rho_{-i}\frac{\delta}{1-\delta}r$$
$$+\frac{\delta\rho_{-i}}{\tau_i(1-\delta\rho)}\left[(1-\rho)\frac{\delta}{1-\delta}r\tau_i - (1-\rho_i)c + [1-(1-\sigma)\rho_i - \rho\sigma]\frac{\delta}{1-\delta}b\tau_i - \tau_i\int_{\alpha_L}^{\overline{\alpha}_i}\alpha f(\alpha)\,d\alpha\right] \quad (14)$$

If we manipulate equation (14), we can see that cutpoint $\overline{\alpha}_i$ is implicitly defined by the following function:

$$\Psi^i \equiv \overline{\alpha}_i(1-\delta\rho) - (1-\delta\rho_{-i})\frac{c}{\tau_i} + [1-\sigma+(\sigma-\delta)\rho_{-i}]\frac{\delta}{1-\delta}b + \delta\rho_{-i}\left(r + \int_{\alpha_L}^{\overline{\alpha}_i}\alpha f(\alpha)\,d\alpha\right) = 0 \quad (15)$$

To see that this function can generate an interior equilibrium, note that:

$$\Psi^i_{\overline{\alpha}_i} = \overline{\alpha}_i[-\delta\rho_{-i}f(\overline{\alpha}_i)] + (1-\delta\rho) + \delta\rho_{-i}\overline{\alpha}_i f(\overline{\alpha}_i) = 1-\delta\rho > 0$$

Because Ψ^i is strictly increasing in $\overline{\alpha}_i$, if there exists a value $\overline{\alpha}_i$ that satisfies $\Psi^i(\overline{\alpha}_i) = 0$, this value is unique. Consider the value of function Ψ^i in the limit as δ becomes arbitrarily small:

$$\lim_{\delta\to 0}\Psi^i = \overline{\alpha}_i - \frac{c}{\tau_i}$$

We can therefore identify the equilibrium cutpoint as δ becomes arbitrarily small:

$$\lim_{\delta\to 0}\Psi^i = 0 \quad \Leftrightarrow \quad \lim_{\delta\to 0}\overline{\alpha}_i = \frac{c}{\tau_i}$$

Recall that by assumption, $\frac{c}{\tau_i} \in (\alpha_L, \alpha_H)$ for every i and α has full support over $[\alpha_L, \alpha_H]$. So player i has a unique interior cutpoint, $\overline{\alpha}_i \in (\alpha_L, \alpha_H)$, for small $\delta > 0$.

Since this argument holds for an arbitrary player i, there exists a Bayesian Nash equilibrium in which equilibrium strategies are implicitly defined by the system of n equations with n endogenous variables $\overline{\alpha} = (\overline{\alpha}_1, \overline{\alpha}_2, \ldots \overline{\alpha}_n)$:

$$\begin{aligned} \Psi^1(\overline{\alpha}) &= 0 \\ \Psi^2(\overline{\alpha}) &= 0 \\ \cdots & \;\cdots\; \cdots \\ \Psi^n(\overline{\alpha}) &= 0 \end{aligned}$$

□

Intermediate Results

Note that Ψ^i is continuously differentiable in all of its arguments. In particular,

$$\Psi^i_{\overline{\alpha}_i} = \overline{\alpha}_i \left[-\delta\rho_{-i} f\left(\overline{\alpha}_i\right)\right] + (1-\delta\rho) + \delta\rho_{-i}\overline{\alpha}_i f\left(\overline{\alpha}_i\right) = 1 - \delta\rho > 0$$

Note that: $\lim_{\delta\to 0} \Psi^i_{\overline{\alpha}_i} = 1$.

By manipulating equation (15) and using substitutions for the ρ-values, we can show that:

$$\Psi^i = \overline{\alpha}_i \left(1 - \delta\rho_j\rho_{-j}\right) - \frac{c}{\tau_i} + (1-\sigma)\frac{\delta}{1-\delta}b + \left(\frac{\rho_j\rho_{-j}}{\rho_i}\right)\left(\delta\frac{c}{\tau_i} + (\sigma-\delta)\frac{\delta}{1-\delta}b + \delta r + \delta\int_{\alpha_L}^{\overline{\alpha}_i} \alpha f(\alpha)\, d\alpha\right)$$

This allows us to see that the derivative of Ψ^i with respect to $\overline{\alpha}_j$ (for $j \neq i$) is:

$$\begin{aligned}
\Psi^i_{\overline{\alpha}_j} &= \overline{\alpha}_i \left[-\delta f\left(\overline{\alpha}_j\right)\rho_{-j}\right] + f\left(\overline{\alpha}_j\right)\left(\frac{\rho_{-j}}{\rho_i}\right)\left(\delta\frac{c}{\tau_i} + (\sigma-\delta)\frac{\delta}{1-\delta}b + \delta r + \delta\int_{\alpha_L}^{\overline{\alpha}_i} \alpha f(\alpha)\, d\alpha\right) \\
&= f\left(\overline{\alpha}_j\right)\rho_{-j}\left[\left(\frac{1}{\rho_i}\right)\left(\delta\frac{c}{\tau_i} + (\sigma-\delta)\frac{\delta}{1-\delta}b + \delta r + \delta\int_{\alpha_L}^{\overline{\alpha}_i} \alpha f(\alpha)\, d\alpha\right) - \delta\overline{\alpha}_i\right] \\
&= \delta f\left(\overline{\alpha}_j\right)\frac{\rho}{\rho_i\rho_j}\left(\frac{c}{\tau_i} + \frac{\sigma-\delta}{1-\delta}b + r + \int_{\alpha_L}^{\overline{\alpha}_i} \alpha f(\alpha)\, d\alpha - \overline{\alpha}_i\rho_i\right)
\end{aligned}$$

Note that: $\lim_{\delta\to 0} \Psi^i_{\overline{\alpha}_j} = 0$.

Other derivatives and arguments about the Jacobian matrix continue to hold. Additionally, we can prove the following intermediate result:

Lemma 2. *In equilibrium, $\overline{\alpha}_i < \frac{c}{\tau_i}$.*

Proof of Lemma 1. By the derivations above, $\Psi^i_{\overline{\alpha}_i} > 0$. Note that:

$$\Psi^i\left(\overline{\alpha}_i = \frac{c}{\tau_i}\right) = \delta\rho_{-i}\left(1-\rho_i\right)\frac{c}{\tau_i} + \left[1 - \sigma + (\sigma-\delta)\rho_{-i}\right]\frac{\delta}{1-\delta}b + \delta\rho_{-i}\left(r + \int_{\alpha_L}^{\frac{c}{\tau_i}} \alpha f(\alpha)\, d\alpha\right)$$

This is positive for small δ. So the equilibrium value of $\overline{\alpha}_i$ that solves $\Psi^i\left(\overline{\alpha}_i\right) = 0$ must be less than $\frac{c}{\tau_i}$. □

General Comparative Statics

The proof of Proposition 2 continues to hold without any alteration.

Proposition 3: When another player's trade stake (τ_j) increases, player i is less likely to file in any given period.

Proof of Proposition 3. The first half of the proof of Proposition 3 continues to hold. Namely:

$$\frac{\partial \overline{\alpha}_1}{\partial \tau_n} = \frac{-\det(\mathbf{C})}{\det(\mathbf{J})} \quad \text{where} \quad \mathbf{C} = \begin{bmatrix} \Psi^1_{\tau_n} & \Psi^1_{\overline{\alpha}_2} & \ldots & \Psi^1_{\overline{\alpha}_{n-1}} & \Psi^1_{\overline{\alpha}_n} \\ \Psi^2_{\tau_n} & \Psi^2_{\overline{\alpha}_2} & \ldots & \Psi^2_{\overline{\alpha}_{n-1}} & \Psi^2_{\overline{\alpha}_n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \Psi^{n-1}_{\tau_n} & \Psi^{n-1}_{\overline{\alpha}_2} & \ldots & \Psi^{n-1}_{\overline{\alpha}_{n-1}} & \Psi^{n-1}_{\overline{\alpha}_n} \\ \Psi^n_{\tau_n} & \Psi^n_{\overline{\alpha}_2} & \ldots & \Psi^n_{\overline{\alpha}_{n-1}} & \Psi^n_{\overline{\alpha}_n} \end{bmatrix}$$

And:

$$\det(\mathbf{C}) = \det\left(\mathbf{C}^{\mathbf{T}}\right) = (-1)^{2n-3}\det(\mathbf{D}) = -\Psi^n_{\tau_n}\det(\mathbf{D_{11}}) = -(1-\delta\rho_{-n})\frac{c}{\tau_n^2}\det(\mathbf{D_{11}})$$

As in the main proof, we define the following matrix to ascertain the sign of $\det(\mathbf{D_{11}})$:

$$\mathbf{E} \equiv \mathbf{D_{11}} = \begin{bmatrix} \Psi^1_{\overline{\alpha}_n} & \Psi^2_{\overline{\alpha}_n} & \ldots & \Psi^{n-1}_{\overline{\alpha}_n} \\ \Psi^1_{\overline{\alpha}_2} & \Psi^2_{\overline{\alpha}_2} & \ldots & \Psi^{n-1}_{\overline{\alpha}_2} \\ \ldots & \ldots & \ldots & \ldots \\ \Psi^1_{\overline{\alpha}_{n-1}} & \Psi^2_{\overline{\alpha}_{n-1}} & \ldots & \Psi^{n-1}_{\overline{\alpha}_{n-1}} \end{bmatrix}$$

$$\begin{aligned} \text{So: } \det(\mathbf{E}) &= \sum_{k=1}^{n-1}(-1)^{k+1}\,\Psi^k_{\overline{\alpha}_n}\det(\mathbf{E_{1k}}) \\ &= \sum_{k=1}^{n-1}(-1)^{k+1}\left[f(\overline{\alpha}_n)\frac{\rho}{\rho_k\rho_n}\left(\delta\frac{c}{\tau_k} + (\sigma-\delta)\frac{\delta}{1-\delta}b + \delta r + \delta\int_{\alpha_L}^{\overline{\alpha}_k}\alpha f(\alpha)\,d\alpha - \delta\overline{\alpha}_k\rho_k\right)\right]\det(\mathbf{E_{1k}}) \\ &= \left[\frac{\delta f(\overline{\alpha}_n)\rho}{\rho_n}\right]\sum_{k=1}^{n-1}(-1)^{k+1}\left(\frac{1}{\rho_k}\right)\left(\frac{c}{\tau_k} + \frac{\sigma-\delta}{1-\delta}b + r + \int_{\alpha_L}^{\overline{\alpha}_k}\alpha f(\alpha)\,d\alpha - \overline{\alpha}_k\rho_k\right)\det(\mathbf{E_{1k}}) \end{aligned}$$

So $\det(\mathbf{E}) > 0$ if and only if the following condition holds:

$$\phi \equiv \sum_{k=1}^{n-1}(-1)^{k+1}\left(\frac{1}{\rho_k}\right)\left(\frac{c}{\tau_k} + \frac{\sigma-\delta}{1-\delta}b + r + \int_{\alpha_L}^{\overline{\alpha}_k}\alpha f(\alpha)\,d\alpha - \overline{\alpha}_k\rho_k\right)\det(\mathbf{E_{1k}}) > 0$$

By the argument in the main proofs regarding the Jacobian matrix, $\lim_{\delta\to 0}\mathbf{E_{11}} = \mathbf{I}$. So $\lim_{\delta\to 0}\det(\mathbf{E_{11}}) = \det(\mathbf{I}) = 1$. For $k = 2, 3, \ldots, n-1$, calculating $\det(\mathbf{E_{1k}})$ requires that we remove the k-th column of $\mathbf{E}$. This removes $\Psi^k_{\overline{\alpha}_k}$ from the k-th row of $\mathbf{E}$. Since all other entries in the k-th row of $\mathbf{E}$ approach 0 as δ approaches 0, $\lim_{\delta\to 0}\det(\mathbf{E_{1k}}) = 0$. So:

$$\lim_{\delta\to 0}\phi = \left(\frac{1}{\rho_1}\right)\left(\frac{c}{\tau_1} - \overline{\alpha}_1\rho_1 + \sigma b + r + \int_{\alpha_L}^{\overline{\alpha}_1}\alpha f(\alpha)\,d\alpha\right)$$

By Lemma 1, we know that $\overline{\alpha}_1 < \frac{c}{\tau_1}$. This implies that $0 < \frac{c}{\tau_1} - \overline{\alpha}_1 < \frac{c}{\tau_1} - \overline{\alpha}_1\rho_1$. So $\lim_{\delta\to 0}\phi > 0$, which implies that $\det(\mathbf{E}) = \det(\mathbf{D_{11}}) > 0$. We can thus conclude that:

$$\det(\mathbf{C}) = -(1-\delta\rho_{-n})\frac{c}{\tau_n^2}\det(\mathbf{D_{11}}) < 0 \quad \Rightarrow \quad \frac{\partial\overline{\alpha}_1}{\partial\tau_n} > 0 \text{ for small } \delta$$

□

The proof of Proposition 4 continues to hold without any alteration.

Diffusion Comparative Statics

Proposition 5: When the number of affected countries increases, each player is less likely to file in any given period.

Proof of Proposition 5. Suppose players have identical trade stakes, $\tau_i = \frac{\tau}{n}$. As in the main proof, our assumption that players have identical trade stakes makes the game symmetric, the system of $\Psi^i(\overline{\alpha})$-equations can be simplified to one equation, which I denote as Ψ^n, with one endogenous variable, $\overline{\alpha}_n$.

Let ρ_n denote the *ex ante* probability that an arbitrary player does not file the dispute when the game has n players. In equilibrium, each player in the n-player game uses cutpoint $\overline{\alpha}_n$, which is implicitly defined by:

$$\Psi^n = \overline{\alpha}_n\left[1-\delta(\rho_n)^n\right] - \left[1-\delta(\rho_n)^{n-1}\right]\frac{cn}{\tau} + \left[1-\sigma+(\sigma-\delta)(\rho_n)^{n-1}\right]\frac{\delta}{1-\delta}b + \delta(\rho_n)^{n-1}\left(r + \int_{\alpha_L}^{\overline{\alpha}_n}\alpha f(\alpha)\,d\alpha\right)$$

Consider the value of function Ψ^n in the limit as δ becomes arbitrarily small:

$$\lim_{\delta\to 0}\Psi^n = \overline{\alpha}_n - \frac{cn}{\tau}$$

We can therefore identify the equilibrium cutpoint as δ becomes arbitrarily small:

$$\lim_{\delta\to 0}\Psi^n = 0 \quad \Leftrightarrow \quad \lim_{\delta\to 0}\overline{\alpha}_n = \frac{cn}{\tau}$$

By the same logic, the unique cutpoint for the $(n+1)$-player game, $\overline{\alpha}_{n+1}$, is implicitly defined by:

$$\begin{aligned}\Psi^{n+1} \;=\; & \overline{\alpha}_{n+1}\left[1-\delta(\rho_{n+1})^{n+1}\right] - \left[1-\delta(\rho_{n+1})^n\right]\frac{c(n+1)}{\tau} \\ & + \left[1-\sigma+(\sigma-\delta)(\rho_{n+1})^n\right]\frac{\delta}{1-\delta}b + \delta(\rho_{n+1})^n\left(r + \int_{\alpha_L}^{\overline{\alpha}_{n+1}}\alpha f(\alpha)\,d\alpha\right)\end{aligned}$$

and the following holds:

$$\lim_{\delta\to 0}\Psi^{n+1} = \overline{\alpha}_{n+1} - \frac{c(n+1)}{\tau} = 0 \quad \Leftrightarrow \quad \lim_{\delta\to 0}\overline{\alpha}_{n+1} = \frac{c(n+1)}{\tau}$$

So $\lim_{\delta\to 0}\overline{\alpha}_n < \lim_{\delta\to 0}\overline{\alpha}_{n+1}$. This means that each player is less likely to file when the number of players increases and δ is small. □

The proof of Propositions 6 and 7 continue to hold without any alteration.

Proposition 8: In observable WTO disputes, cases that challenge diffuse policies will, on average, have more legal merit than cases that challenge concentrated policies.

Proof of Proposition 8. Suppose players have identical trade stakes, $\tau_i = \frac{\tau}{n}$. By the Proof of Proposition 1, the marginal benefit for player i of type α_i from filing the case when there are n players is:

$$\begin{aligned}\Psi^n(\alpha_i) \quad = \quad & \alpha_i\left[1-\delta(\rho_n)^n\right]-\left[1-\delta(\rho_n)^{n-1}\right]\frac{cn}{\tau} \\ & +\left[1-\sigma+(\sigma-\delta)(\rho_n)^{n-1}\right]\frac{\delta}{1-\delta}b+\delta(\rho_n)^{n-1}\left(r+\int\limits_{\alpha_L}^{\alpha_i}\alpha f(\alpha)\,d\alpha\right)\end{aligned}$$

So:

$$\Psi^n_r(\alpha_i)=\delta(\rho_n)^{n-1}>0 \quad \text{and} \quad \lim_{r\to\infty}\Psi^n(\alpha_i)=\infty>0$$
$$\text{and} \quad \lim_{\delta\to 0}\Psi^n(\alpha_i|r=0)=\alpha_i-\frac{cn}{\tau}<0\Leftrightarrow\alpha_i<\frac{cn}{\tau}$$

So for small δ and very high α_i-values, player i always files the case, regardless of the value of r. But for small δ and low α_i-values, the intermediate value theorem ensures that there exists a unique critical value $\overline{r}(\alpha_i,n)>0$ such that $\Psi^n(\alpha_i|\overline{r}(\alpha_i,n))=0$, meaning that a player of type α_i files if and only if $\overline{r}(\alpha_i,n)\leq r$.

Also note that the marginal benefit for player i of type α_i from filing the case when there are $n+1$ players is:

$$\begin{aligned}\Psi^{n+1}(\alpha_i) \quad = \quad & \alpha_i\left[1-\delta(\rho_{n+1})^{n+1}\right]-\left[1-\delta(\rho_{n+1})^{n}\right]\frac{c(n+1)}{\tau} \\ & +\left[1-\sigma+(\sigma-\delta)(\rho_{n+1})^{n}\right]\frac{\delta}{1-\delta}b+\delta(\rho_{n+1})^{n}\left(r+\int\limits_{\alpha_L}^{\alpha_i}\alpha f(\alpha)\,d\alpha\right)\end{aligned}$$

Given the logic above, for small δ and low α_i-values there exists a unique critical value for the $(n+1)$-player game, $\overline{r}(\alpha_i,n+1)>0$, such that $\Psi^{n+1}(\alpha_i|\overline{r}(\alpha_i,n+1))=0$.

Finally, note that:

$$\lim_{\delta\to 0}\left[\Psi^n(\alpha_i)-\Psi^{n+1}(\alpha_i)\right]=\frac{c}{\tau}>0$$

By definition of the critical value:

$$\Psi^n(\alpha_i|\overline{r}(\alpha_i,n))=0=\Psi^{n+1}(\alpha_i|\overline{r}(\alpha_i,n+1))<\Psi^n(\alpha_i|\overline{r}(\alpha_i,n+1)) \quad \text{for small } \delta$$

This implies that $\overline{r}(\alpha_i,n)<\overline{r}(\alpha_i,n+1)$. So as the number of players increases, larger values of r will be necessary for a player of type $\alpha_i<\frac{cn}{\tau}$ to file the case. □

A.8 Implication: Diffuseness and Total Trade Stake in Observed Cases

Recall that for an n-player game, the equilibrium cutpoint, x, is defined by:

$$\Psi^n(x) \equiv x\left[1-\delta F(x)^n\right] - \left[1-\delta F(x)^{n-1}\right]\left(\frac{cn}{\tau} - \frac{\delta}{1-\delta}b\right) + \delta F(x)^{n-1}\left[r + \int_{\alpha_L}^{x} \alpha f(\alpha)\, d\alpha\right]$$

Then:

$$\frac{\partial \Psi^n(x)}{\partial \tau} = \left[1-\delta F(x)^{n-1}\right]\frac{cn}{\tau^2} > 0$$

So the cutpoint, $\overline{\alpha}_n$, is decreasing in the total trade stake. This means that each player is more likely to file as the total trade stake increases. Since diffuseness makes each player less likely to file, conditional on being filed, the total trade stake in a diffuse case should be larger than in a concentrated case.